MOAA 2022: Accuracy Round

October 8th, 2022

Rules

- No mathematical texts, notes, or online resources of any kind are permitted. Rely on your brain!
- Compasses, protractors, rulers, straightedges, graph paper, blank scratch paper, and writing implements are generally permitted, so long as they are not designed to give an unfair advantage.
- No computational aids (including but not limited to calculators, phones, calculator watches, and computer programs) are permitted on any portion of the MOAA.
- No individual may receive help from any other person, including members of their team. Consulting any other individual is grounds for disqualification.

How to Compete

- **In Person:** After completing the test, write your answers down in the provided Accuracy Round answer sheet. The proctors will collect your answer sheets immediately after the test ends.
- **Online:** After completing the test, you should input your answers, along with your Team ID and name, into the provided Accuracy Round Google Form.

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Accuracy Round Problems

Welcome the Accuracy Round! The Accuracy Round consists of 10 problems, ordered in approximately increasing difficulty, to be solved in 50 minutes. All answers are nonnegative integers no larger than 1,000,000.

A1. **[4]** Find the last digit of 2022^{2022}.

A2. **[4]** Let $a_1 < a_2 < \ldots < a_8$ be eight real numbers in an increasing arithmetic progression. If $a_1 + a_3 + a_5 + a_7 = 39$ and $a_2 + a_4 + a_6 + a_8 = 40$, determine the value of a_1.

A3. **[5]** Patrick tries to evaluate the sum of the first 2022 positive integers, but accidentally omits one of the numbers, N, while adding all of them manually, and incorrectly arrives at a multiple of 1000. If adds correctly otherwise, find the sum of all possible values of N.

A4. **[6]** A machine picks a real number uniformly at random from $[0, 2022]$. Andrew randomly chooses a real number from $[2020, 2022]$. The probability that Andrew's number is less than the machine's number is $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

A5. **[8]** Let $ABCD$ be a square and P be a point inside it such that the distances from P to sides AB and AD respectively are 2 and 4, while $PC = 6$. If the side length of the square can be expressed in the form $a + \sqrt{b}$ for positive integers a, b, then determine $a + b$.

A6. **[10]** Positive integers $a_1, a_2, \ldots a_{20}$ sum to 57. Given that M is the minimum possible value of the quantity $a_1!a_2!\ldots a_{20}!$, find the number of positive integer divisors of M.

A7. **[13]** Jessica has 16 balls in a box, where 15 of them are red and one is blue. Jessica draws balls out the box three at a time until one of the three is blue. If she ever draws three red marbles, she discards one of them and shuffles the remaining two back into the box. The expected number of draws it takes for Jessica to draw the blue ball can be written as a common fraction $\frac{m}{n}$ where m, n are relatively prime positive integers. Find $m + n$.

A8. **[16]** The *Lucas sequence* is defined by these conditions: $L_0 = 2, L_1 = 1$, and $L_{n+2} = L_{n+1} + L_n$ for all $n \geq 0$. Determine the remainder when $L_{2019}^2 + L_{2020}^2$ is divided by L_{2023}.

A9. **[16]** Let $ABCD$ be a parallelogram. Point P is selected in its interior such that the distance from P to BC is exactly 6 times the distance from P to AD, and $\angle APB = \angle CPD = 90°$. Given that $AP = 2$ and $CP = 9$, the area of $ABCD$ can be expressed as $m\sqrt{n}$ where m and n are positive integers and n is not divisible by the square of any prime. Find $m + n$.

A10. **[18]** Consider the polynomial $P(x) = x^{35} + \cdots + x + 1$. How many pairs (i, j) of integers are there with $0 \leq i < j \leq 35$ such that if we flip the signs of the x^i and x^j terms in $P(x)$ to form a new polynomial $Q(x)$, then there exists a nonconstant polynomial $R(x)$ with integer coefficients dividing both $P(x)$ and $Q(x)$?